**Portfolio Optimization Using Evolutionary Computation**

**Abstract:**

Computational Intelligence is applied in the financial field as an increasingly popular technology. As one main cluster, Evolutionary Computation is regards as one effective Optimization tool in finance. In particular, one of the most abundant applications is portfolio selection. This essay presents fundamental composition of the intelligent financial portfolio, and some meta-heuristic approach to portfolio optimization problem, such as Particle Swarm Optimization (PSO), Genetic Relation Algorithm (GRA) and Genetic Network Programming (GNP). Then the experiment results, range of application and limitations are compared and analyzed comprehensively and the state of problem and further studies in the future are summarized finally.

**1. Introduction**

There are many factors affecting the prices of financial assets. However, because the effects might be non-linear, time-lagged and non-stationary, the modeling and trading of financial markets are challenging. Despite these difficulties one family of computational algorithms, so-called Natural Computing, becomes the very effective problem-solver, which includes Neurocomputing, Evolutionary Computing, Social Computing, Immunocomputing, Physical Computing, and Developmental & Grammatical Computing.

Based on neo-Darwinian principles of evolution, Evolutionary Computation (EC) is one main cluster of natural computing. High-quality solutions as subsequent generation are created by parallel search with a population of individuals and fitness-based selection, just like natural selection and survival of the fittest in biology.

As an especially powerful form of EC, Genetic Programming (GP) can allow the solution to be co-evolved with parameters during the evolutionary process, which offers the particular utility in financial modeling, since the circumstance is always data rich but theory poor. Except Model Induction, another key category of applications of EC in Finance is Optimization, such as portfolio selection.

A financial portfolio consists of a diverse set of financial assets, also called securities or investments, like stocks, bonds, futures, CFDs or ETFs, in order to maximize return on a certain and accepted degree of risk, i.e. the chance of incurring in a loss. The original and standard formulation of the portfolio selection problem is Markowitz’s Mean–Variance (M–V) model, the expectation and variance (or the standard deviation) of the return are used as evaluation criteria for return and risk as follows:

Subject to:

Where is the expected return of the asset, represents the percentage of the total investment on that it is allocated over the asset, and the risk measure is associated with each asset, represented by . The goal of the Markowitz model is to find an optimal portfolio  of assets such that the is maximized while minimizing the (variance of the return). This formulation produces a quadratic programming problem and there are a number of studies on Portfolio Optimization exist to tackle this using Evolutionary Computation.

The rest of the essay is organized as follows. Section 2 describes the composition of an intelligent portfolio system. In Section 3, some unconventional methods for the portfolio optimization problem, such as PSO, GRA and GNP, are summarized. Final conclusions and future research are drawn in Section 4.

**2.** **Intelligent** **Financial Portfolio Composition**

In order to select the most profitable assets without the skills of financial market’s specialists, a potential system is proposed based on known intelligent computation techniques, in particular Evolutionary Algorithms, which aims to manage a financial portfolio by using technical analysis indicators.

**2.1** **Portfolio Theory and Market Analysis**

Portfolio management is the core work of financial portfolio, which includes the act of deciding which assets should be selected, when and how much capital should be invested or removed in term of investors’ risk preferences. In order to realize the automatic management of a portfolio, two distinct forms of management could be applied: passive management and active management. The latter aiming to buy undervalued stocks and sell overvalued ones can possibly guarantee higher profitability levels.

For the sake of reducing the unsystematic risk, diversification technique is applied widely, which means mixing a variety of investment vehicles to reduce the chance of losing capital.

When evaluating the market and selecting potential profitable assets, fundamental analysis and technical analysis can be applied exclusively or jointly. In detail, fundamental analysis focuses on the intrinsic value of a particular company, including overall economy, industrial conditions and financial situation. The value is crucial to find the potential profit compared with the current security’s price.

Following the premise that market action includes all the fundamental information, technical analyst believes only the volume of transactions and the securities prices are needed to study for forecasting market behavior. And the kind of analysis usually demonstrates technical indicators, like Simple moving average (SMA). In practical, both strategies could coexist, for example, fundamental analysis is used to pick proper companies then technical analysis for deciding buy and sell signals.

**2.2** **Classical Optimization Solution**

According to Modern Portfolio Theory (MTP), the rational investors should reduce the risk and increase the expected returns by diversifying their investments. Several techniques are employed to compute an efficient combination of the expected return and the variance, concretely, focus on computing the Efficient Frontier (EF, aka Pareto Frontier in Markowitz’s model). Once the frontier is found, the final choice of portfolio is determined by the investor’s risk preference. However, we have to take into account several restrictions like transaction costs or industrial regulations when we apply the model in the real world. In addition, the means of estimating expected return are proposed compared with Mean–Variance in Markowitz’s model, because investors are more concerned with the total return of the portfolio than risk.

**2.2.1** **Single-objective Evolutionary Algorithms (SOEA)**

The single-objective optimization problem is solved by using a trade-off function relating minimizing risk and maximizing the return of the portfolio. The original approximation comes from Loraschy et al in 1993. The two objectives can be parametrized to yield a convex combination programming problem:

Then many models are proposed with some new risk measures and additional realistic constrains. A variety of metaheuristics like GAs, simulated annealing (SA) and tabu search (TS) are used, the accomplished tests demonstrate they are versatile enough, not requiring any modification, in case of considering other risk measures or constraints.

In regards to the representation of portfolio, two distinct lists are utilized at first. One is the set of assets included in the portfolio (Q) and another with the respective investment allocation (S), such as following example:

Q = {AMZN, GOOG} S = {0.5, 0.5}

In 2008, Aranha and Hitoshi proposed a brand new representation using a tree-based structure as follows. The distinct form accelerates the evolution of a good solution and increases the portfolio’s understandability.



**2.2.2** **Multi-objective Evolutionary Algorithms**

Streichert et al. (2003) put forward Multi-objective Evolutionary Algorithms (MOEAs), and their goal was to optimize two conflicting objectives, maximizing return and minimizing risk. Besides three real world constraints, namely cardinality constraints, buy-in thresholds and round-lots are considered:

Cardinality constraints: the maximum and minimum number of assets that a portfolio manager wants to include in the portfolio;

Buy-in thresholds: the lower proportion limit specified for each security;

Round-lots: the number of any asset included in the portfolio must be multiple of normal trading lot (100).

As the extension of EA, Memetic Algorithm (MA) is employed to perform a local search to convert an infeasible solution to a feasible one respecting the considered constrains before evaluating a population in order to refine its individuals. Also the Lamarckism mechanism could updates the decision variables, i.e. the percentages of allocation investment that then can be inherited to the next generation.

Unlike through a trade-off function that relates risk and return in SOEAs, MOEAs tent to rank solutions evaluating risk and return separately as two objectives, which means a set of solutions are offered in a single run.

**2.2.3** **Genetic Programming (GP) Approach Based on Technical Analysis**

The approach performing a market evaluation based on technical analysis can reward us with a greater profitability since overvalued and undervalued stocks to produce profit will be found. Genetic Programming (GP) technique is applied by Liad Wagman in 2003. The initial stocks and percentage allocation of portfolios are assigned randomly, and every portfolio is evaluated through six technical rules responsible for generating buy or not-buy signals and weights:

|  |  |  |  |
| --- | --- | --- | --- |
| **Risk**  **(60%)** | 1. Today’s price-average price of the previous 12 trading days **(15%)** | **Moving Average Rules** | Generate buy signals if equations (1) and (2) are greater than zero, respectively. |
| 2. Today’s price-average price of the previous 50 trading days **(15%)** |
| 3. Today’s price-maximum price of the previous 5 trading days **(15%)** | **Trading Range Breakout Rules** | Generate buy signals if equations (3) and (4) are greater than zero, respectively. |
| 4. Today’s price-maximum price of the previous 50 trading days **(15%)** |
| **Return**  **(40%)** | 5. Today’s price-minimum price of the previous 5 trading days **(20%)** | **Filter Rules** | Generate buy signals if today’s price has risen 1% in respect to the minimum of previous 5 or 63 days, respectively. |
| 6. Today’s price-minimum price of the previous 63 trading days **(20%)** |

Note that the model above only aims to maintain a specific portfolio without considering the management issues.

**2.3** **Solution’s Architecture**

António et al. proposed the architecture of the portfolio management system as following diagrams, which includes five fundamental modules. They are User Presentation Module, Financial Data Processing Module, Optimization Module, Investment Simulator Module and Technical Rules Module, and the Optimization Module corresponds to the main core of the application. The GA optimization technique is utilized to perform the analysis.

****

**2.3.1 Initial Generation**

Each individual or chromosome is represented by a real valued array structure consisting weights and four bound values. Every weight element is given in term of the importance of a specific technical rule, like Exponential Moving Average (EMA), the Hull Moving Average (HMA), the Rate of Change (ROC), the Double Crossover Method, the Relative Strength Index (RSI), the Moving Average Convergence Divergence (MACD), the True Strength Index (TSI), and the On Balance Volume (OBV). Surely more technical indicators could be easily extended in the algorithm. And the bound values define the necessary score that equity needs to obtain:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 1st Rule | 2nd Rule | … | Last rule | Buy limit | Short limit | Close buy position | Close short position |
| [0, 1] | [0, 1] | … | [0, 1] | [0, 1] | [-1, 0] | [-1, 1] | [-1, 1] |

In order to guarantee the initial generation randomly generated is feasible, the sum of all weights should equal to one, the following normalization is applied:

The classifier is defined by the following equation:

Where is the score given by the technical rule to stock .

After the optimization performed by the algorithm, the classifier is updated, where the technical indicators are balanced, then all the assets within the market are classified. The stocks whose classification is higher than the value given by the Buy Limit field adopt long positions. The ones whose classification is below the Short Limit adopt short positions. The last two bound values; Close Buy Position and Close Short Position determine the necessary score to achieve so a specific position in the portfolio can be closed.

**2.3.2 Selection**

After selecting the encoded representation it is time to decide how the specific algorithm choose the individuals that will generate offspring for the next generation. Truncation Selection methodology could be performed to sort the population according to fitness. Then a roulette procedure is applied on the set of best individuals to decide the breeders. By default, half of the population is set as parents.

**2.3.3 Mutation**

The number of variables to be mutated depends on Mutation Rate, chromosome size and the number of population individual. If there is only one individual, the elitism is maintained.

**2.3.4 Crossover**

In respect to the crossover procedure, one-cut point methodology offers the best result for the represented chromosome. From the randomly selected crossover point, all variables are swapped between both parents, and inherited by two new individuals.

**2.3.5 Evaluation Function**

When the efficiency of different investments during a particular period needs to be evaluated, the Return on Investment (ROI) function is applied. In SOEA, the goal of the model is to maximize the ROI, while the risk i.e. volatility of returns can be involved simultaneously in MOEAs.

**2.4 Performance Evaluation**

The results achieved by the solution during the years of 2003 to the first semester of 2009 for the DJI index, and the period of 2006 to the first semester of 2009 for the S&P500 index demonstrates when market crashes or runs sideways, the genetic approach offers a much more powerful and robust solution for all market conditions. And if the market is rapidly rising, Buy and hold (B&H) procedure should be better choice.

**3. Unconventional methods for** **the portfolio optimization problem**

The problem of most popular heuristic optimization techniques GA focuses on the chromosomes, for example many chromosomes are coded into the same portfolio, or similar chromosomes may be coded into very different portfolios. It causes more difficulties for GA to produce better chromosomes from good ones. Also these problems multiply the GA’s search space and make GA less efficient in finding the optimal portfolio. Moreover GP occasionally causes some bloating problems for its tree structure. In order to overcome these drawbacks, some unconventional methods are applied for the portfolio optimization problem.

**3.1** **Particle Swarm Optimization (PSO)**